Modulations of Aharonov-Bohm oscillations in a quantum box

1989 J. Phys.: Condens. Matter 1 10207

(http://iopscience.iop.org/0953-8984/1/50/022)

J. Phys.: Condens. Matter 1 (1989) 10207–10210. Printed in the UK

LETTER TO THE EDITOR

Modulations of Aharonov–Bohm oscillations in a quantum box

M Yosefin† and M Kaveh‡§
† Department of Physics, Bar-Ilan University, Ramat-Gan 52100, Israel
‡ Cavendish Laboratory, University of Cambridge, Cambridge, UK

Received 23 October 1989

Abstract. We produce a simple model that leads to modulations of the Aharonov–Bohm oscillations in the weak and strong coupling limit. Our model produces a complete explanation of all the magneto-transport phenomena in a quantum box that were recently observed by Brown *et al*.

Recent experiments by Wharam *et al* [1], van Wees *et al* [2] and particularly by Brown *et al* [3] demonstrated the advantages of using GaAs–AlGaAs heterostructures in the investigation of the Aharonov–Bohm (AB) effect. Due to the high mobility of the two-dimensional electron gas in GaAs devices, both the elastic and the inelastic mean free paths can be larger than the size of the region where quantum interference takes place. This absence of random scattering eliminates the resistance fluctuations and the Al'tshuler, Aronov and Spivak [4] $h/2e$ oscillations which can mask the AB effect in normal metal rings [5]. This enables one to study the AB effect even in the *weak* coupling limit. Indeed, the first observations of the AB oscillations in the tunnelling regime were made by using the GaAs heterostructures [1–3]. Surprisingly, the AB h/e oscillations observed in the experiment [3] were modulated by a low-frequency oscillation of a new type. The purpose of this letter is to discuss the origin of this new modulation. We show that the modulations extend into two regimes, the weak and the strong coupling limits.

A simple constriction exhibiting the AB oscillations consists of a wide region of arbitrary shape connected to two narrow leads (figure 1). Such a constriction can be defined by biasing a Schottky gate of an appropriate shape. Negative gate bias leads to the creation of narrow conducting channels whose width can be adjusted continuously by varying the gate voltage. Simultaneously, the gate voltage reduces the density of the two-dimensional electron gas in the channels.

In a sufficiently strong perpendicular magnetic field, the bulk Landau levels are localised and the current is carried by the edge states $\psi_n(x, y)$. As a result of the depletion, the number of current carrying edge channels in the leads is smaller than in the reservoirs. The main feature of such a constriction is the plateau-like behaviour of the resistance as function of the gate voltage. The origin of the plateaux results from the conductance being proportional to the number of transmitted edge states; thus, when an edge state is reflected the resistance jumps by h/e^2. In the experiment of Brown *et al* [3], six edge

§ Permanent address: Department of Physics, Bar-Ilan University, Ramat-Gan 52100, Israel.

0953-8984/89/5010207 + 04 $02.50

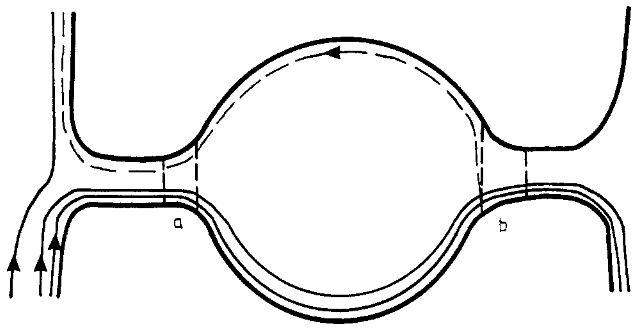

Figure 1. Schematic diagram of the quantum box model. The potential barriers at a and b are represented by vertical broken lines. The full curves represent incident edge states and the broken curves represent edge states reflected from the potential barriers.

states are expected. They result from the spin splitting of three Landau levels. In the range of gate voltage used in the experiment, the first four plateaux were observed. This description assumes either perfectly transmitted or reflected edge states. In practice this assumption is never met experimentally. In the experiment of Brown *et al* a potential barrier is present at the entrance and exit of the constriction (points a and b in figure 1), which leads to deviations from perfect transmission. The resistance was measured as a function of the gate voltage in the presence of a strong magnetic field. In addition to the expected plateau-like structure, it was found that the resistance oscillates as a function of the gate voltage. Usually, in previous experiments, these oscillations were associated with the AB effect. Surprisingly, the amplitude of the oscillations was modulated periodically rather than being uniform. Detailed analysis of the data yielded two periodic behaviours of the resistance: the first period is inversely proportional to the square of the gate voltage, and the other period is inversely proportional to the gate voltage. This structure was found to appear between each pair of plateaux, and also in the plateau regime only above the third plateau.

In this letter we propose a model that is capable of explaining all the above phenomena. In particular, we show that the additional period found experimentally arises from an amplitude modulation rather than an interference effect.

We model the constriction as a quantum box in which the geometrical confinement of the 'box' is unimportant, and which is attached to narrow leads of width $w < R$ (R is the size of the box). At the entrance and exit from the quantum box there are potential barriers $V(x, y)$. We first consider weak potentials, which are relevant to the structure found on the plateaux. These potentials lead to reflections of the edge states at *both* sides of the quantum box and the total reflectance will depend on the interference of the two waves.

Consider an edge state propagating from left to right (figure 1). The potential at point a will reflect part of the wave. The transmitted wave will again be reflected and transmitted by the potential at b. The reflected waves from b and from a (see broken curves in figure 1) will interfere. The total reflected wave in the lead can be written as

$$\psi = \sum_{mn} (A^{(a)}_{mn} + A^{(b)}_{mn} e^{i\phi_{mn}})\varphi_n(y) e^{-ik_n x} \tag{1}$$

where $\varphi_n(y)\exp(-ik_n x)$ is the wave function of the nth reflected edge state [6], $A^{(a,b)}$

are the reflection amplitudes from state m to state n and ϕ_{mn} is the phase acquired by an edge state in the quantum box. In the weak scattering limit, A_{mn} is given by

$$A_{mn}^{(a,b)} = \langle \varphi_n(-y)e^{-ik_nx} | V^{(a,b)}(x,y) | \varphi_m(y)e^{ik_mx}\rangle. \tag{2a}$$

Assuming that $V^{(a,b)}(x,y)$ is a potential barrier of range L and height $V_0^{(a,b)}$ we obtain

$$A_{mn}^{(a,b)} = \langle \varphi_n(-y) | \varphi_m(y)\rangle [V_0^{(a,b)}/\mathrm{i}(k_m + k_n)] \sin(k_m + k_n)(L/2). \tag{2b}$$

The overlap between two edge states decays exponentially with the width of the wire, so only the edge state with the slowest decay will dominate. Using this fact, the reflectance R can be written as

$$R = |A^{(a)}|^2 + |A^{(b)}|^2 + 2\mathrm{Re}(A^{(a)}A^{(b)}) \cos \phi_n. \tag{3}$$

This leads to an additional resistance, $(h/e^2)R$, due to the reflection at the potential barriers. We now study the dependence of R on the gate voltage. Equation (3) reveals *two* dependencies on the gate voltage: a *phase* dependence, and an *amplitude* dependence. We first consider the phase dependence. The additional phase acquired by the edge state is approximately proportional to the area of the quantum box. This dependence leads to the AB oscillations which have been widely observed. The new ingredient here is that the amplitude itself depends on the gate voltage and is responsible for the new effect that was observed. By varying the gate voltage one changes the range L of the potential barrier which causes a periodic modulation. From equations (2) and (3) it follows that the oscillatory part of the resistance is proportional to $\sin^2 k_n L \cos \theta_n$. This explains the main features of the experiment. The phase is much more sensitive to changes in the gate voltage since it depends on the area of the quantum box. This leads to the rapid AB oscillations which are now slowly modulated by the slower periodic dependence of the amplitude on the gate voltage.

The amplitude depends exponentially on the width of the wire. Therefore for wide wires the above effect should disappear. Indeed, for small gate voltages (which correspond to a wide wire) a pure plateau-like structure without oscillations was observed. As the gate voltage was increased the two types of oscillation revealed themselves.

We now consider the structure found by Brown *et al* between the plateaux. In this regime the gate voltage is such that the potential barriers at the entrance and exit of the quantum box are strong for a particular edge state, and the current is caused by tunnelling. In this limit, the structure of the conductance is determined mainly by the states *in* the box which are weakly coupled to the wires. In particular, the AB oscillations are formed whenever an edge state inside the box matches the Fermi energy. Indeed, in the experiments, AB oscillations were observed between the plateaux. This is unlike the situation in a strongly disordered structure where the AB oscillations due to tunnelling are suppressed. The interesting feature of the experiment of Brown *et al* is that even in this regime the same type of modulations of the AB oscillations persist. Our explanation of this phenomenon is as follows. There are two ingredients that determine the current. The first is the strength of the potential barrier which determines the particular edge state that will tunnel through the wire, and the second is the explicit form of the eigenstates in the box and in the wire. The matching of these two types of eigenstate will determine the tunnelling rate. The eigenstates in the quantum box can be written as [7]

$$\psi_{\mathrm{QB}}(r,\theta) = e^{im\theta} f_{ml}(r). \tag{4}$$

For simplicity we use the magnetically unperturbed eigenstates within the wire

$$\psi_w = \sin(n\pi y/\omega)\, e^{ik_n x}. \tag{5}$$

The tunnelling rate of the nth edge state is given by [8]

$$|\alpha_n \Gamma_n/(E_F - E_n + i\Gamma_\phi)|^2 \tag{6}$$

where Γ_n is the energy width of the state n and $|\alpha_n| \leq 1$. As the gate voltage is changed, the states pass through the Fermi energy which causes the AB oscillations due to the denominator in (6). The Γ_n in the numerator is (as we shall show below) responsible for the slower modulations of the AB oscillations. Γ_n depends exponentially on the width of the potential barrier and therefore only the edge state with the smallest decay length will contribute to the current. This tunnelled edge state must match the eigenstates in the wire as given by equation (5). By matching the two states we find

$$\Gamma_m \simeq \sin(m\omega/R) \tag{7}$$

which implies a conductance proportional to $\sin^2(m\omega/R)$. Since ω and R are *linearly* dependent on the gate voltage, this explains the slow modulation of the AB oscillations in the weak coupling limit as found by Brown *et al* [3].

In summary, we have proposed a simple model for the modulation of the AB oscillations in both the weak and the strong coupling limits. In particular, our model explains the recent experimental results for the extra structure observed in magneto-transport through a quantum box.

We are grateful to M Pepper, R J Brown and E G Smith for stimulating discussions regarding this work. We also acknowledge the hospitality of the Cavendish Laboratory. Moshe Kaveh is grateful for a Royal Society Visiting Fellowship.

References

[1] Wharam D A, Pepper M, Newbury R, Ahmed H, Hasko D G, Peacock D C, Frost, J E F, Ritchie D A and Jones G A 1989 *J. Phys.: Condens. Matter* **1** 3369
[2] van Wees B J, Kourvenhoven L P, Harmans C J P M, Williamson J G, Timmering C E, Brockaart M E I, Foxon C T and Harris J J 1989 *Phys. Rev. Lett.* **62** 2523
[3] Brown R J, Smith C G, Pepper M, Kelly M J, Newbury R, Ahmed H, Hasko D G, Frost J E F, Peacock D C, Ritchie D A and Jones G A C 1989 *J. Phys.: Condens. Matter* **1** 6286
[4] Al'tshuler B L, Aronov A G and Spivak B Z 1981 *Pis. Zh. Eksp. Teor. Fiz.* **33** 101 (Engl. Transl. *JETP Lett.* **33** 94)
[5] Washburn S and Webb R A 1986 *Adv. Phys.* **35** 375
[6] van Houten H, Beenakker C W J, Williamson J G, Brockaart M E I, van Loosdrecht P H M, van Wees B J, Mooij J E, Foxon C T and Harris J J 1989 *Phys. Rev.* B **39** 8556
[7] Halperin B I 1982 *Phys. Rev.* B **25** 2185
[8] Buttiker M, Imry Y and Azbel M Ya 1984 *Phys. Rev.* A **30** 1982